Reflection of light from semi-infinite turbid media

Alexander A. Kokhanovsky

Institute of Particle Technology and Environmental Engineering, Technical University of Clausthal, Leibnizstrasse 19, D-38678 Clausthal-Zellerfeld, Germany, and Stepanov Institute of Physics, National Academy of Sciences of Belarus, 68 F. Scarina Avenue, 220072 Minsk, Belarus

Received December 2, 1997; revised manuscript received July 10, 1998; accepted July 13, 1998

The simple equation for the spherical albedo of a semi-infinite turbid medium is obtained. The accuracy of the approximation was studied with the numerical solution of the radiative transfer equation. The error is smaller than 10% for water clouds in the visible and the near infrared. © 1998 Optical Society of America [S0740-3232(98)00111-2]

OCIS codes: 290.7050, 120.5700, 030.5620.

Studies of light reflection from semi-infinite turbid media are important for many applications, including medical imaging and cloud, snow, and foam optics. The spherical albedo of such media R can be obtained by integration of the reflection function $r(\mu, \mu_0, \psi)$ (Ref. 1):

$$R = \frac{2}{\pi}\int_0^1 \mu \mathrm{d}\mu \int_0^1 \mu_0 \mathrm{d}\mu_0 \int_0^{2\pi} r(\mu, \mu_0, \psi)\mathrm{d}\psi, \qquad (1)$$

where $\mu = \cos \nu$; $\mu_0 = \cos \nu_0$; ν and ν_0 are the observation and the incidence angles, respectively; and ψ is the azimuth. The function $r(\mu, \mu_0, \psi)$ is the solution of the radiative transfer equation.[1] This solution depends on the single scattering albedo ω and the phase function $p(\theta)$, where θ is the scattering angle. The value of ω represents the probability that a photon injected into a turbid medium will be scattered after interaction with a particle. The function $p(\theta)$ is the probability that a photon will scatter in the direction specified by the scattering angle θ. According to the similarity principle,[1] turbid media that have different phase functions $p(\theta)$ but the same value of the asymmetry parameter

$$g = \frac{1}{2}\int_0^{\pi} p(\theta)\cos\,\theta\,\sin\,\theta d\,\theta \qquad (2)$$

and the single scattering albedo ω have approximately the same radiative characteristics. Thus in the first coarse approximation the value of R depends only on two parameters, namely, ω and g.

An approximate formula for the spherical albedo of semi-infinite turbid media was obtained by Perelman *et al.*[2] within the framework of the path integral formalism:

$$R = \exp\left[-\alpha\left(\frac{\ln\frac{1}{\omega}}{1-g}\right)\right]^{1/2}, \qquad (3)$$

where, according to Perelman *et al.*,[2] the value of α is a function of g.

The purpose of this paper is to show that the value of α should be constant and to compare Eq. (3) with exact radiative transfer calculations.

The following exact asymptotical relation for the spherical albedo of a semi-infinite medium can be obtained from the radiative transfer theory as $\omega \to 1$ (Ref. 1):

$$R = 1 - 4\left[\frac{1-\omega}{3(1-g)}\right]^{1/2}. \qquad (4)$$

This relation is valid for any phase functions and values of g. It is possible to find the value of α in Eq. (3) by comparing Eqs. (4) and (3) as $\omega \to 1$. Indeed, it follows from Eq. (3) as $\omega \to 1$ that

$$R = 1 - \alpha\left(\frac{1-\omega}{1-g}\right)^{1/2}, \qquad (5)$$

where the asymptotical formula

$$\lim_{\omega\to 1}\left[\ln\left(\frac{1}{\omega}\right)\right] = 1 - \omega \qquad (6)$$

is used. Thus one can see that the value of α does not depend on g and that

$$\alpha = \frac{4}{\sqrt{3}}. \qquad (7)$$

The value of α in the paper by Perelman *et al.*[2] was approximately 3.1 at $g = 0.908$ and approximately 2.9 at $g = 0.757$. This value is nearly 30% larger than it should be according to Eq. (7). It follows from Eqs. (3) and (7) that

$$R = \exp\left\{-4\left[\frac{\ln\frac{1}{\omega}}{3(1-g)}\right]^{1/2}\right\}. \qquad (8)$$

One can see that

$$\lim_{\omega\to 0} R(\omega) = 0, \quad \lim_{\omega\to 1} R(\omega) = 1, \qquad (9)$$

as expected. The dependence $R(\omega)$, calculated with Eq. (8) at $g = 0.85$, is represented in Fig. 1. It is interesting that at $\omega > 0.9$ ($R > 0.2$) a somewhat simpler formula for calculation of the spherical albedo can be applied [see Eq. (6) and Fig. 1]:

 © 1998 Optical Society of America

$$R = \exp\left\{-4\left[\frac{1-\omega}{3(1-g)}\right]^{1/2}\right\}. \quad (10)$$

Equation (10) was obtained by Rozenberg[3] for a special case of weakly absorbing media. A comparison of the results of the spherical-albedo calculation obtained with Eq. (8) and with a numerical solution of the radiative transfer equation[1] for semi-infinite water clouds with the gamma particle size distribution

$$f(a) = Ba^6 \exp(-1.5a), \quad (11)$$

where B is the normalization constant $[\int_0^\infty f(a)\mathrm{d}a = 1]$, is presented in Fig. 2. One can see that the accuracy of the approximate Eq. (8) is high.

In conclusion, a simple approximation (8) was proposed to calculate the spherical albedo of turbid semi-infinite media. This approximation can be used to avoid complex numerical calculations of the total reflectance R and absorption $A = 1 - R$ of radiation by semi-infinite thick turbid media.

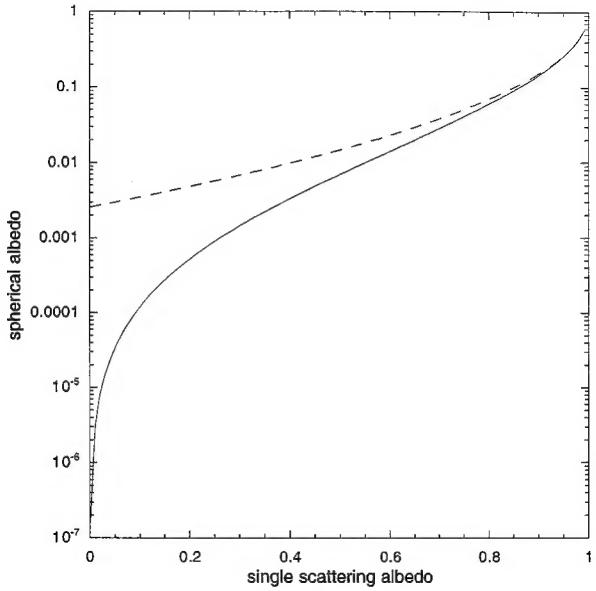

Fig. 1. Dependence of the spherical albedo of a semi-infinite medium on the single scattering albedo at $g = 0.85$, calculationed with Eq. (8) (solid curve) and with Eq. (10) (dashed curve).

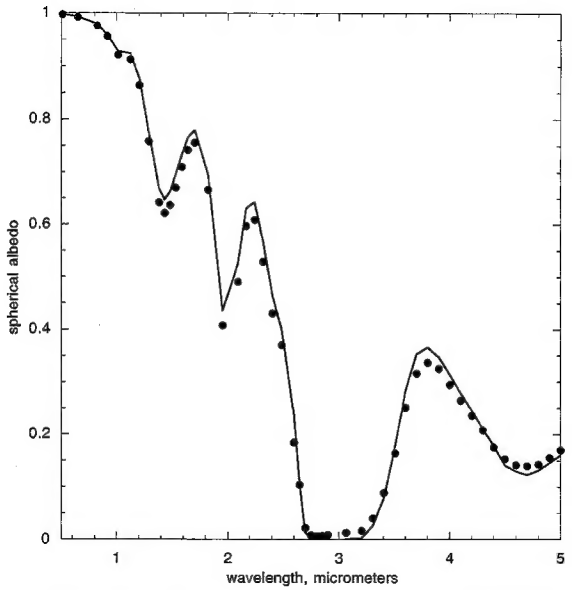

Fig. 2. Dependence of the spherical albedo of a semi-infinite water cloud with particle size distribution (11) on the wavelength, calculated with Eq. (8) (solid curve) and with the numerical solution of the radiative transfer equation (symbols).

ACKNOWLEDGMENT

This research was supported by the Alexander von Humboldt Foundation.

REFERENCES

1. E. P. Zege, A. P. Ivanov, and I. L. Katsev, *Image Transfer through a Scattering Medium* (Springer-Verlag, Berlin, 1991).
2. L. T. Perelman, J. Wu, I. Itzkan, and M. S. Feld, "Photon migration in turbid media using path integrals," Phys. Rev. Lett. **72**, 1341–1343 (1994).
3. G. V. Rozenberg, "Optical characteristics of thick weakly absorbing scattering layers," Dokl. AN SSSR **145**, 775–777 (1962).